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Superconducting π qubit with three Josephson junctions

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We propose a qubit consisting of a superconducting ring with two zero junctions and one ferromagnetic π junction. In the system, degenerate states appear in the phase space without an external magnetic field because of a competition between the zero and π states. Quantum tunneling between the degenerate states leads to a formation of bonding and antibonding states which are used as a bit. For manipulating the states, only a small magnetic field around zero is required. This feature leads to a large-scale integration and a construction of the smaller-sized qubit which is robust to the decoherence by external noises. © *2006 American Institute of Physics*. [DOI: 10.1063/1.2189191]

Quantum computing has attracted a great deal of interest in recent years.[1] As a candidate for an elemental unit of a quantum computer (qubit), many proposals have been carried out, e.g., photons, ion traps, and nuclear spins. Among the proposals, solid-state devices have great advantages in large-scale integration and flexibility of layout, but still face the problem of reducing the decoherence effect due to their coupling to the environment. Recently, there are many works on superconducting qubits in which the freedom of the superconducting phase is utilized as a bit (flux qubit).[2–12] The three-junction type of the flux qubit proposed by Mooij *et al.*[2] consists of a superconducting loop with three Josephson junctions, and the bonding and antibonding states formed by applying an external magnetic field are used as a bit.[2–8] For this qubit, single-qubit rotation (Rabi oscillation), the direct coupling between the two qubits, and the entangled states have been demonstrated.[5–8] A qubit with zero and ferromagnetic π junctions, which requires no external magnetic field for the formation of the coherent two states, has been proposed.[9] As another flux qubit, there are "quiet" qubits consisting of *s*-wave/*d*-wave superconducting junctions or five Josephson junctions including one ferromagnetic π junction.[10,11] In the quiet qubit, no current flows (quiet) in the system during the operation and therefore it is expected to be robust to the decoherence by the environment. On the other hand, the difficulty in the realization of the quiet qubits has been pointed out because these qubits need complicated manipulations as well as a system consisting of many junctions including *d*-wave or ferromagnetic π junctions.[12]

Furthermore, recent advances in microfabricating techniques have promoted extensive work on spin electronics.[13,14] In particular, ferromagnetic π junctions have been studied actively as a superconducting spin-electronic device.[15–24] In a superconductor/ferromagnet (SC/FM) junction, the superconducting order parameter penetrates into FM and oscillates due to the exchange field in FM.[17] The system is stable at the phase difference equal to π when the order parameters in two SCs take a different sign in a SC/FM/SC junction, and this state is called the "π state." The π state is also explained from the point of view of spin-split Andreev bound states.[20] There are several experimental observations of the π state in junction-type and superconducting quantum interference device (SQUID)-type structures.[18–24] In the junction-type geometries, cusp structures in the dependences of the critical current on the temperature and the FM's thickness have been observed as evidence of the transition between the π state and the ordinary zero state.[18–21] More recently, the π state has been confirmed via the reversed current-phase relation.[21] In SQUID-type structures consisting of a superconducting ring with zero and π junctions, it has been reported that the magnetic-field dependence of the critical current is π shifted compared to that in the ring with two zero junctions.[22,23] In a superconducting ring with a single π junction, an asymmetry in a magnetic-field dependence of a spontaneous current due to the π state has been observed.[24]

In this letter, we propose a qubit with two ordinary zero junctions and one ferromagnetic π junction. We show that the qubit does not require an external magnetic field for forming the coherent two states, and only a small external field is needed for distinguishing the states. This feature makes it possible to construct the qubit with a smaller size, and therefore this qubit is advantageous to large-scale integration and is expected to have a long decoherence time.

We consider a superconducting ring with three Josephson junctions as shown in Fig. 1. The two Josephson junctions are ordinary zero junctions with the phase differences θ_1, θ_2, and the other is a ferromagnetic π junction with the

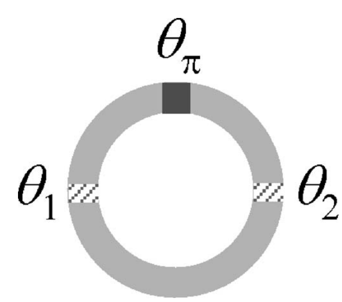

FIG. 1. Schematic diagram of a π qubit consisting of a superconducting ring with two ordinary zero junctions and a π junction. The phase differences at the zero junctions and the π junction are $\theta_{1(2)}$ and θ_π, respectively.

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phase difference θ_π. The Hamiltonian in this system is expressed as $H=T+U$, where T is an electrostatic energy term written as

$$T=-\frac{4E_{C0}}{E_{C\pi}+2E_{C0}}\times\left[(E_{C\pi}+E_{C0})\left(\frac{\partial^2}{\partial\theta_1^2}+\frac{\partial^2}{\partial\theta_2^2}\right)-2E_{C0}\frac{\partial^2}{\partial\theta_1\theta_2}\right], \quad (1)$$

where $E_{C0(\pi)}=e^2/2C_{0(\pi)}$ is the Coulomb energy for a single charge at the zero (π) junction, $C_{0(\pi)}$ is the capacitance of the zero (π) junction, and e is the elementary charge. The term U indicates a potential energy expressed as

$$U=-E_0(\cos\theta_1+\cos\theta_2)-E_\pi\cos(\theta_\pi+\pi)+\frac{(\Phi-\Phi_{\text{ext}})^2}{2L_s}, \quad (2)$$

where the Josephson coupling constant is E_0 in the zero junctions and E_π in the π junction, Φ is the total flux in the ring, Φ_{ext} is the external flux, and L_s is the self-inductance of the ring. Because of a single-valued wave function around the ring, the total flux and the phase differences satisfy the relation $\theta_1+\theta_2+\theta_\pi=2\pi\Phi/\Phi_0-2\pi l$, where Φ_0 is the unit flux and l is an integer. Substituting the relation in Eq. (2), we obtain

$$U=-E_0(\cos\theta_1+\cos\theta_2)+E_\pi\cos\left(2\pi\frac{\Phi}{\Phi_0}-\theta_1-\theta_2\right)+\frac{(\Phi-\Phi_{\text{ext}})^2}{2L_s}. \quad (3)$$

From the condition that U is minimum with respect to Φ, i.e., $\partial U/\partial\Phi=0$, we obtain the self-consistent equation as follows:

$$\alpha\beta\sin\left(2\pi\frac{\Phi}{\Phi_0}-\theta_1-\theta_2\right)=2\pi\left(\frac{\Phi}{\Phi_0}-\frac{\Phi_{\text{ext}}}{\Phi_0}\right), \quad (4)$$

where $\alpha=E_\pi/E_0$ and $\beta=4\pi^2E_0L_s/\Phi_0^2$. By solving Eq. (4) numerically, we obtain $\Phi=\Phi(\theta_1,\theta_2)$ as a function of θ_1 and θ_2. Substituting the obtained $\Phi(\theta_1,\theta_2)$ in the expression for U [Eq. (3)], we get $U=U(\theta_1,\theta_2)$ as a function of θ_1 and θ_2. Throughout this letter, we assume $E_{C\pi}=E_{C0}/\alpha$ and use $\alpha=0.8$, $\beta=3.0\times10^{-3}$. The value of α is controllable by changing the contact area and the thickness of both the insulators and the ferromagnet. The value of β is reasonable for the micrometer-size ring and the Josephson junction with several hundred nanoampere of the critical current.

Figures 2(a) and 2(b) show the θ_1, θ_2 dependence of U without an external magnetic flux ($\Phi_{\text{ext}}=0$). As shown in Fig. 2, U has degenerate two states $|\uparrow\rangle$ near $(\theta_1/\pi,\theta_2/\pi)=(-1/2+2m,-1/2+2n)$ and $|\downarrow\rangle$ near $(1/2+2m,1/2+2n)$ in the phase space where m and n are integers. At the $|\uparrow\rangle$ and $|\downarrow\rangle$ states, the currents of magnitude $\approx0.8I_0$ with clockwise and anticlockwise directions flow in the ring, respectively, where I_0 is the critical current in the zero junctions. Because of quantum tunneling between the degenerate $|\uparrow\rangle$ and $|\downarrow\rangle$ states, bonding $|0\rangle\propto|\uparrow\rangle+|\downarrow\rangle$ and antibonding $|1\rangle\propto|\uparrow\rangle-|\downarrow\rangle$ states that are used as a quantum bit are formed. Both at the $|0\rangle$ and $|1\rangle$ states, no current flows because the $|\uparrow\rangle$ and $|\downarrow\rangle$ components with the equivalent weight exist in the both states. The energy gap ΔE between the $|0\rangle$ and $|1\rangle$ states appears due to the quantum tunneling, and the existence of these states is confirmed by the microwave resonance with the frequency $\nu=\Delta E/h$, where h is the Planck constant. From the numerical calculation for $E_0/E_{C0}=30$, $\Delta E\approx2.9\times10^{-24}$ J, which corresponds to the frequency $\nu\approx4.4$ GHz. This value of ν is comparable to that for the three-junction qubit.[2–8] Figures 3(a) and 3(b) show the θ_1, θ_2 dependence of U within an external magnetic flux $\Phi_{\text{ext}}=0.05\Phi_0$. As shown in Fig. 3, the degeneracy of the $|\uparrow\rangle$ and $|\downarrow\rangle$ states is lifted due to the finite external magnetic flux. In this case, the $|\uparrow\rangle$ component increases and the $|\downarrow\rangle$ component decreases in the bonding $|0\rangle$ state, and vice versa in the antibonding $|1\rangle$ state. Therefore, finite spontaneous currents with the clockwise and anticlockwise directions flow at the $|0\rangle$ and $|1\rangle$ states, respectively, and the corresponding magnetic fluxes are induced in the ring. One can distinguish the states of the qubit through the measurements of the spontaneous flux by a SQUID located around the ring. As shown above, the qubit incorporating the π junction does not requires an external magnetic field for the formation of the coherent two states, and requires only a small field around zero for detecting the states. A required field for the manipulation is of the order of a millitesla even if the dimension of the qubit is several hundred nanometers. By this feature, (i) the qubit is advantageous to a large-scale integration, and (ii) it becomes easier to construct a smaller size of the qubit, which is robust to the decoherence effect due to the coupling to the environments. The π qubit with the two Josephson junctions proposed in Ref. 9 requires a

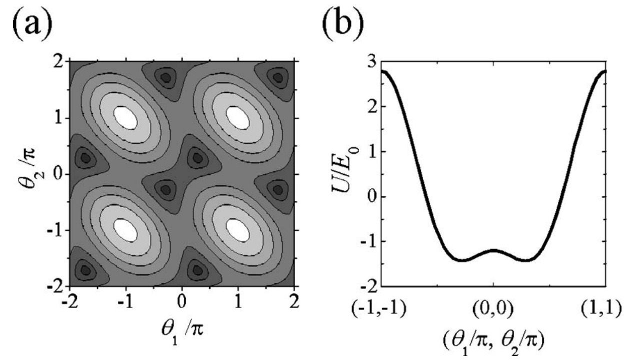

FIG. 2. (a) Contour plot of U/E_0 in the phase space without an external magnetic flux ($\Phi_{\text{ext}}=0$). The lighter and darker parts correspond to the larger and smaller values of U/E_0, respectively. (b) The phase dependence of U/E_0 in the diagonal direction from $(\theta_1/\pi,\theta_2/\pi)=(-1,-1)$ to (1,1).

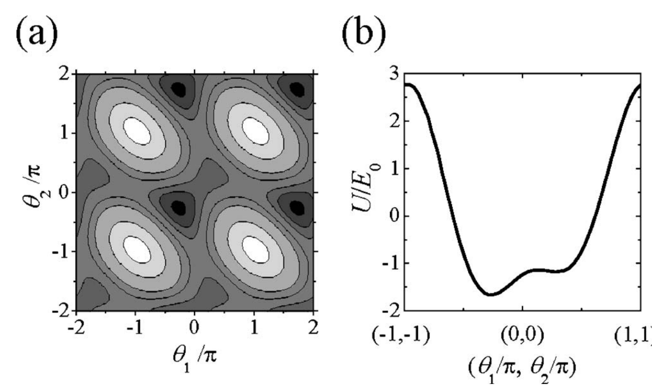

FIG. 3. (a) Contour plot of U/E_0 in the phase space with an external magnetic flux $\Phi_{\text{ext}}=0.05\Phi_0$. The lighter and darker parts correspond to the larger and smaller values of U/E_0, respectively. (b) The phase dependence of U/E_0 in the diagonal direction from $(\theta_1/\pi,\theta_2/\pi)=(-1,-1)$ to (1,1).

metallic-contact π junction, whereas the present qubit works when the interface of the π junction is insulating as well as metallic due to the two zero junctions.

For the quantum computation, a construction of a universal gate is needed. Most popular configuration of the universal gate consists of single-qubit rotation and controlled-NOT (CNOT) gates. The single-qubit rotation gate is done by applying a microwave with the frequency $\nu = \Delta E/h$ (Rabi oscillation). The CNOT gate is also realized as follows. Here we consider "qubit A" as a control bit and "qubit B" as a target bit under a small external magnetic field. Because of the magnetic interaction between the spontaneous fluxes in the two qubits, the energy gap in qubit B depends on the state of qubit A and is expressed as $\Delta E_{\mathrm{B0(B1)}}$ when qubit A is in the $|0\rangle$ ($|1\rangle$) state. By applying the microwave with the frequency $\Delta E_{\mathrm{B1}}/h$ to qubit B, the state of qubit B changes through the Rabi oscillation only when qubit A is in the $|1\rangle$ state.

In summary, we have proposed a qubit consisting of a superconducting ring with two zero junctions and a single π junction. In the system, the potential energy has double minima in the phase space without external magnetic fields because of the competition between the zero and π states. The bonding and antibonding states (coherent states) are formed due to the quantum tunneling between the two degenerate states, and the coherent states are used as a bit in the qubit. A small external magnetic field around zero is needed for manipulating the state of the qubit. These features lead to a large-scale integration and a smaller size of the qubit that is resistant to the decoherence by the external noise and has a long decoherence time.

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